ARTICLE NO. 78-572

Attitude Behavior of a Dual-Spin Spacecraft Composed of Asymmetric Bodies

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This paper deals with an attitude motion of a dual-spin spacecraft composed of asymmetric bodies. First, the attitude stability of the dual-spin spacecraft is investigated by the method of averaging. It is clarified that the dual-spin spacecraft has an unstable region of attitude motion due to the asymmetry of the bodies. Criteria for the onset of the unstable motion are established. Next, nonstationary attitude motions of the dual-spin spacecraft through the unstable region are examined by an analytical method that utilizes the WKB method. It is demonstrated that, under certain conditions, a trap phenomenon (inability to pass through the unstable region) may occur. Analytical criteria for the onset of the trap phenomenon are established. These results are compared with those of digital computer simulations.

Introduction

A DUAL-SPIN spacecraft consists of two rigid bodies connected by means of a bearing axis. One of the bodies, the rotor, is spun at a high rate to stabilize the spacecraft. The second body, the platform, is nearly despun so that its payload instrument can be inertially pointed. Because of the immediate application to communications satellites, the investigation of the attitude motion of a dual-spin spacecraft has received considerable attention. There exists much literature on the effect of energy dissipations in the bodies on the attitude motion and stability of a dual-spin spacecraft.[1-4] A problem area of equal importance is the effect of the asymmetry on the attitude behavior of a dual-spin spacecraft.[3-6] Despite the importance of the problem, many of the attitude dynamics questions on the subject seem to remain unanswered. The purpose of this paper is to discuss the attitude behavior of a dual-spin spacecraft composed of asymmetric bodies. First, the attitude stability of this type of spacecraft is examined. When the spin velocity of either of the two bodies approaches the angular velocity of a nutational body motion, it is possible for the dual-spin spacecraft to be unstable in its pure rotation about the axis of rotation and permit a parametric excitation of a nutational body motion.

Now, a dual-spin spacecraft must have the capability of despinning the platform so that its mission instruments are inertially fixed. During the course of the despin maneuver, this type of spacecraft will pass through the unstable region. The nonstationary attitude behavior of the dual-spin spacecraft in passing through the unstable region is studied. Under certain conditions, the spacecraft cannot pass through the unstable region and a trap phenomenon will occur. Scher and Farrenkopf have also discussed a trap phenomenon that was encountered while despinning the platform.[5] The trap phenomenon treated here is different from the trap phenomenon treated in Ref. 5. The latter phenomenon requires an asymmetry on one body and a dynamic imbalance on the other and occurs when the nutation rate of the asymmetric body equals the relative spin rate. On the other hand, the former requires only asymmetry on the rotor and occurs when body nutation rate of the rotor vanishes.

Equations of Motion

The dual-spin spacecraft chosen for this study is shown schematically in Fig. 1. The system consists of two asymmetric bodies A and B. A relative motion between bodies A and B is restricted to rotation about a common axis. The mass centers of bodies A and B lie on the axis of rotation, and the axis of rotation is a common principal axis of the two bodies. Let us introduce two sets of reference axes (A_1, A_2, A_3) and (B_1, B_2, B_3). The reference axes (A_1,A_2, A_3) and (B_1, B_2, B_3) are fixed along the principal axes of body A and B, respectively. The origins of the two reference axes are fixed on the mass center C of the spacecraft. The A_3 and B_3 axes coincide with the axis of rotation. The angular velocity of the body A is denoted by (ω_1, ω_2, ω_3) in the reference axes (A_1, A_2, A_3). The relative rotation of body B with respect to body A is denoted by angle ϕ. Then, equations of motion for the system in a free space are written in the form

$$
\begin{aligned}
I\dot{\omega}_1 &+ [(I_{A_3}-I)\omega_3 + I_{B_3}\Gamma]\omega_2 \\
&= -\Delta_A(\dot{\omega}_1+\omega_3\omega_2) + \Delta_B\Big[-\frac{\mathrm{d}}{\mathrm{d}t}(\omega_1\cos 2\phi+\omega_2\sin 2\phi) \\
&\quad + (\omega_1\sin 2\phi - \omega_2\cos 2\phi)\omega_3\Big]
\end{aligned} \tag{1}
$$

$$
\begin{aligned}
I\dot{\omega}_2 &- [(I_{A_3}-I)\omega_3 + I_{B_3}\Gamma]\omega_1 \\
&= \Delta_A(\dot{\omega}_2-\omega_3\omega_1) + \Delta_B\Big[-\frac{\mathrm{d}}{\mathrm{d}t}(\omega_1\sin 2\phi-\omega_2\cos 2\phi) \\
&\quad - (\omega_1\cos 2\phi + \omega_2\sin 2\phi)\omega_3\Big]
\end{aligned} \tag{2}
$$

$$I_{A_3}\dot{\omega}_3 = 2\Delta_A\omega_1\omega_2 \tag{3}$$

$$I_{B_3}\dot{\Gamma} = -\Delta_B[(\omega_1^2-\omega_2^2)\sin 2\phi - 2\omega_1\omega_2\cos 2\phi] + \eta \tag{4}$$

$$\dot{\phi} = \Gamma - \omega_3 \tag{5}$$

where

$$I = (I_{A_1}+I_{A_2})/2 + (I_{B_1}+I_{B_2})/2$$

$$\Delta_A = (I_{A_1}-I_{A_2})/2$$

$$\Delta_B = (I_{B_1}-I_{B_2})/2$$

Presented as Paper 78-572 at the AIAA 7th Communications Satellite Systems Conference, San Diego, Calif., April 24-27, 1978; submitted April 28, 1978; revision received Oct. 4, 1978.

Index category: Spacecraft Dynamics and Control.

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Γ is the component of the angular velocity of body B along the B_3 axis; I_{A_1}, I_{A_2}, I_{A_3} are the moments of inertia of body A about A_1, A_2, A_3 axes, respectively; I_{B_1}, I_{B_2}, I_{B_3} are the moments of inertia of body B about B_1, B_2, B_3 axes, respectively; and η is the sum of motion and friction torque applied to body B about B_3 axis.

Attitude Stability

Let us now investigate the effect of asymmetry of bodies on the attitude stability of a dual-spin spacecraft. We shall suppose here that $\eta=0$; the motor torque is designed to counteract a bearing friction only. The asymmetry of bodies contributes something of particular importance when the following conditions are satisfied:

$$\text{Case 1} \qquad (I_{A_3}-I)\omega_3+I_{B_3}\Gamma\simeq 0 \tag{6}$$

$$\text{Case 2} \qquad I_{A_3}\omega_3+(I_{B_3}-I)\Gamma\simeq 0 \tag{7}$$

Case 1 corresponds to the case where the spin velocity of body A is near the angular velocity of a nutational body motion, whereas case 2 corresponds to the case where the spin velocity of body B is near the angular velocity of a nutational body motion. First, let us study case 1. Introduce new variable a by the equation

$$\omega_1=a+a^* \qquad \omega_2=-i(a-a^*) \tag{8}$$

where a^* is a complex conjugate of a. Substitution of Eq. (8) into Eqs. (1-5) leads to

$$\dot a=i[(I_{A_3}-I)\omega_3+I_{B_3}\Gamma]a-(\Delta_A/I)(\dot a^*+i\omega_3 a^*)$$
$$-(\Delta_B/I)[\dot a^*+i(2\Gamma-\omega_3)a]e^{i2\phi} \tag{9}$$

$$\dot\omega_3=-2i(\Delta_A/I_{A_3})(a^2-a^{*2}) \tag{10}$$

$$\dot\Gamma=-2i(\Delta_B/I_{B_3})(a^2-a^{*2})e^{i2\phi} \tag{11}$$

$$\dot\phi=\Gamma-\omega_3 \tag{12}$$

Let us suppose here that the parameters Δ_A/I and Δ_B/I are small. Then, we can conclude from Eqs. (9-12) that the variables a, ω_3, and Γ are slowly varying functions because the right-hand sides of Eqs. (9), (10), and (11) are small, while the variable ϕ varies relatively rapidly. Hence, solutions of Eqs. (9-12) are established by using the method of averaging applied to a system containing both slow and rapid motions.[7] The first approximate solutions can be obtained as follows: First, Eq. (12) is solved with a, ω_3, and Γ being constant:

$$\phi=(\Gamma-\omega_3)t \tag{13}$$

Then, substituting this solution into Eqs. (9), (10), and (11) and averaging over the period $\pi/(\Gamma-\omega_3)$, we obtain the first approximation equations for a, ω_3, and Γ in the form

$$\dot a=i[(I_{A_3}/I-1)\omega_3+(I_{B_3}/I)\Gamma]a-i(\Delta_A/I)\omega_3 a^* \tag{14a}$$

$$\dot\omega_3=-2i(\Delta_A/I_{A_3})(a^2-a^{*2}) \tag{14b}$$

$$\dot\Gamma=0 \tag{14c}$$

Equations (14) can be found to have a steady-state solution, which corresponds to a pure rotation:

$$a=a_0 \tag{15a}$$

$$\omega_3=\omega_{3_0} \qquad \text{(a constant)} \tag{15b}$$

$$\Gamma=\Gamma_0 \qquad \text{(a constant)} \tag{15c}$$

Then, let us consider the stability of the steady-state solution. Perturbed motions in the neighborhood of the solution are expressed in the form

$$a=\delta a_r+i\delta a_i \qquad \omega_3=\omega_{30}+\delta\omega_3 \qquad \Gamma=\Gamma_0+\delta\Gamma \tag{16}$$

where δa_r, δa_i, $\delta\omega_3$, and $\delta\Gamma$ are perturbations. Substituting Eqs. (16) into Eqs. (14) and neglecting higher terms, we obtain the nontrivial variational equations

$$\delta\dot a_r=[(I_{A_3}/I-1)\omega_{30}+(I_{B_3}/I)\Gamma_0]\delta a_i-(\Delta_A/I)\omega_{30}\delta a_i \tag{17a}$$

$$\delta\dot a_i=-[(I_{A_3}/I-1)\omega_{30}+(I_{B_3}/I)\Gamma_0]\delta a_r-(\Delta_A/I)\omega_{30}\delta a_r \tag{17b}$$

The characteristic equation of Eqs. (17) is as follows:

$$S^2+[(I_{A_3}/I-1)\omega_{30}+(I_{B_3}/I)\Gamma_0]^2-(\Delta_A/I)^2\omega_{30}^2=0 \tag{18}$$

Stability requires that roots of the characteristic equation must not have a positive real part:

$$[(I_{A_3}/I-1)+(I_{B_3}/I)(\Gamma_0/\omega_{30})]^2>(\Delta_A/I)^2 \tag{19}$$

For case 2 the calculations can be carried out in a similar manner. The necessary and sufficient condition for the stability of a pure rotation is derived as

$$[(I_{A_3}/I)(\omega_{30}/\Gamma_0)+(I_{B_3}/I-1)]^2>(\Delta_B/I)^2 \tag{20}$$

When condition (19) is not fulfilled, a pure rotation becomes unstable due to the asymmetry of body A, while instability of pure rotation due to the asymmetry of body B will occur when condition (20) is not fulfilled. Conditions (19) and (20) are always met in the absence of asymmetry; i.e., $\Delta_A=\Delta_B=0$.

Next, let us consider the behavior of the unsteady attitude motion of the spacecraft, which is established as a result of the instability of a pure rotation. We shall here confine ourselves to unsteady motions of the spacecraft, which correspond to the instability due to the asymmetry of body A. Introducing new variables R and θ by the equation

$$a=Re^{i\theta} \tag{21}$$

and substituting into Eqs. (14), we obtain

$$\dot R=-(\Delta_A/I)R\omega_3\sin 2\theta \tag{22}$$

$$\dot\theta=[(I_{A_3}/I-1)\omega_3+(I_{B_3}/I)\Gamma]-(\Delta_A/I)\omega_3\cos 2\theta \tag{23}$$

$$\dot\omega_3=4(\Delta_A/I)R^2\sin 2\theta \tag{24}$$

$$\dot\Gamma=0 \tag{25}$$

From Eq. (25)

$$\Gamma=\Gamma_0 \tag{26}$$

Using Eqs. (22) and (24), we obtain, upon integration, an integral of motion

$$R^2+\tfrac{1}{4}\omega_3^2=C_0 \tag{27}$$

where $C_0=R_i^2+\tfrac{1}{4}\omega_{3i}^2$ and R_i and ω_{3i} are initial values of R and ω_3, respectively. Another integral of motion follows from Eqs. (23, 24, 26, and 27):

$$(I_{A_3}/I)(I_{A_3}/I-1)\omega_3^2/2+(I_{A_3}I_{B_3}/I^2)\Gamma_0\omega_3$$
$$+2(\Delta_A/I)[C_0-(I_{A_3}/I)\omega_3^2/4]\cos 2\theta=C_1 \tag{28}$$

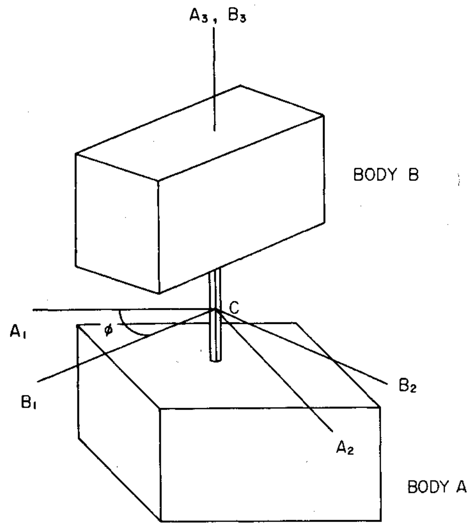

Fig. 1 Spacecraft configuration.

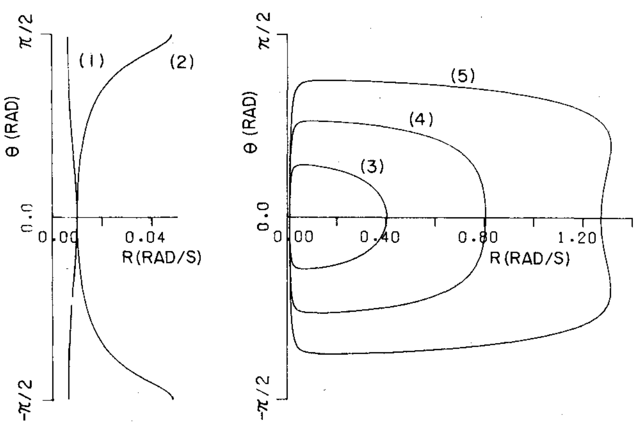

Fig. 2 Trajectories of the system in $R\theta$ plane.

where

$$C_I = (I_{A_3}/I)(I_{A_3}/I - 1)\omega_{3i}^2/2 + (I_{A_3}I_{B_3}/I^2)\Gamma_0\omega_{3i}$$
$$+ 2(\Delta_A/I)[C_0 - (I_{A_3}/I)\omega_{3i}^2/4]\cos 2\theta_i$$

and θ_i is an initial value of θ. Figure 2 shows some examples of the curve (28) where the system parameters are $I_{A_3}/I = 0.8$, $I_{B_3}/I = 0.4$, $\Delta_A/I = 0.1$, and $R_0 = 0.01$ rad/s:

1)	$\omega_{3i} = 1.0$ rad/s	$\Gamma_i = 1.0$ rad/s	(stable case)
2)	$\omega_{3i} = 4.5$ rad/s	$\Gamma_i = 1.0$ rad/s	(stable case)
3)	$\omega_{3i} = 1.5$ rad/s	$\Gamma_i = 1.0$ rad/s	(unstable case)
4)	$\omega_{3i} = 2.0$ rad/s	$\Gamma_i = 1.0$ rad/s	(unstable case)
5)	$\omega_{3i} = 3.0$ rad/s	$\Gamma_i = 1.0$ rad/s	(unstable case)

In the stable cases small initial values of R lead to relatively small maximum values of R. On the other hand, in the unstable cases small initial values of R build up to larger values. The profiles of R show a sinusoidal form with long period.

Nonstationary Attitude Motion through the Unstable Region

Let us now investigate the nonstationary attitude behavior of the spacecraft passing through the unstable region. We shall here confine ourselves to the nonstationary attitude motion of the spacecraft passing through the unstable region due to the asymmetry of body A. In this case the asymmetry of body B has little effect on the attitude motion; the asymmetry of body B may be ignored. We shall suppose here that $\Delta_A > 0$. Furthermore, we shall suppose that the torque η is small. Neglecting the terms containing the asymmetry of body B in Eqs. (1-5), we obtain

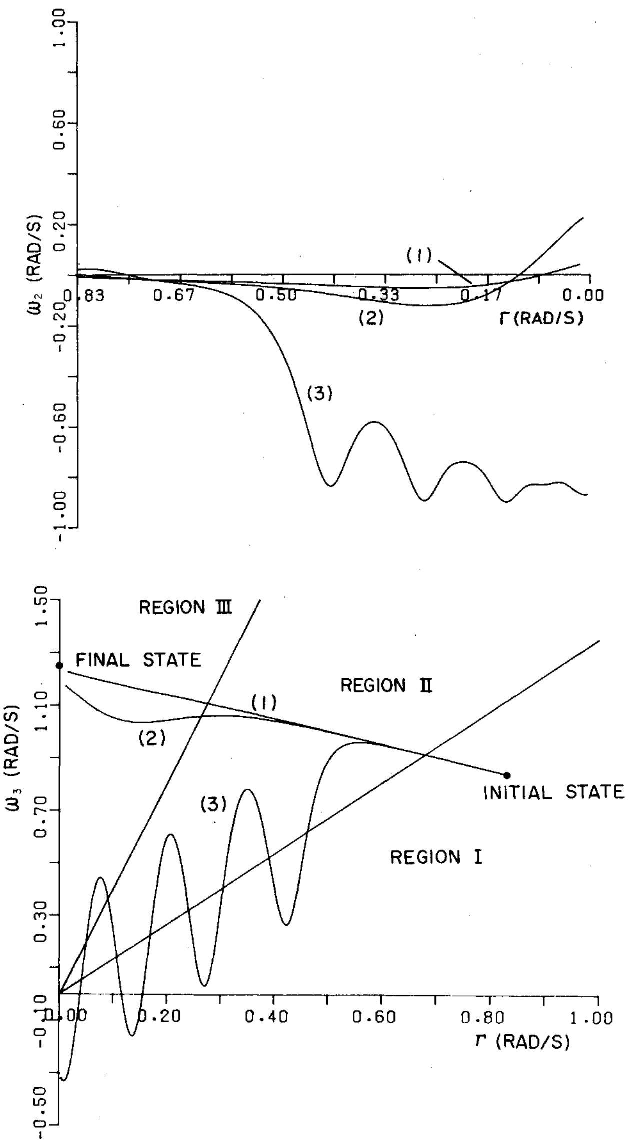

Fig. 3 Nonstationary attitude motion through unstable region.

$$I\dot{\omega}_I + [(I_{A_3} - I)\omega_3 + I_{B_3}\Gamma]\omega_2 = -\Delta_A(\dot{\omega}_I + \omega_3\omega_2) \quad (29)$$

$$I\dot{\omega}_2 - [(I_{A_3} - I)\omega_3 + I_{B_3}\Gamma]\omega_I = \Delta_A(\dot{\omega}_2 - \omega_3\omega_I) \quad (30)$$

$$I_{A_3}\dot{\omega}_3 = 2\Delta_A\omega_I\omega_2 \quad (31)$$

$$\dot{h} = \eta \quad (32)$$

where $h = I_{B_3}\Gamma$, the component of angular momentum of body B along the B_3 axis.

Now, let $\eta < 0$, so that we have the despin maneuver of body B. The despin maneuver of body B corresponds to a movement of the system in Fig. 3b from initial state to final state. Let us suppose that the system starts at some point in the stable region (region I). As h decreases, the system reaches a region of instability (region II); further decrease of h brings the system out of this region (region III). A nutational body motion is at first damped. Then, reaching region II, it begins to be amplified. This continues until region III is reached, whereupon the nutational body motion is again damped. We begin by examining small nutational body motions; i.e., the quantities ω_1 and ω_2 are small. From Eq. (32)

$$h=\eta t+h_i \tag{33}$$

where h_i is an initial value of ω_3. Substituting Eqs. (33) and (34) into Eqs. (29) and (30), we obtain

$$\omega_3=-\eta t/I_{A_3}+\omega_{3i} \tag{34}$$

where ω_{3i} is an initial value of ω_3. Substituting Eqs. (33) and (34) into Eqs. (29) and (30), we obtain

$$\dot{\omega}_1+\alpha_1\omega_2=0 \tag{35a}$$

$$\dot{\omega}_2-\alpha_2\omega_2=0 \tag{35b}$$

where

$$\alpha_1=\alpha_{10}(h-h_1) \qquad \alpha_2=\alpha_{20}(h-h_2)$$
$$\alpha_{10}=(1-\Delta_A)/[(1+\Delta_A)I_{A_3}]$$
$$\alpha_{20}=(1+\Delta_A)/[(1-\Delta_A)I_{A_3}]$$
$$h_1=[1-I_{A_3}/(1-\Delta_A)]H_0 \qquad h_2=[1-I_{A_3}/(1+\Delta_A)]H_0$$
$$H_0=I_{A_3}\omega_{3i}+h_i$$

Equations (35) become linear ordinary difference equations with slowly varying coefficients. In order to obtain asymptotic solutions of Eqs. (35), we will employ the method of multiple scales.[8] The time scales, θ_1, θ_2, and h are introduced:

$$\frac{d}{dt}\theta_1=\lambda_1 \qquad \frac{d}{dt}\theta_2=\lambda_2 \qquad h=\eta t+h_i \tag{36}$$

where λ_i are the roots of $\lambda^2+\alpha_1\alpha_2=0$.

In addition, we assume that an asymptotic expansion of the solution of Eqs. (35) in the form

$$\omega_1=\sum_{n=0}^{\infty}\eta^n A_{1n}e^{\theta_1}+\sum_{n=0}^{\infty}\eta^n A_{2n}e^{\theta_2} \tag{37a}$$

$$\omega_2=\sum_{n=0}^{\infty}\eta^n B_{1n}e^{\theta_1}+\sum_{n=0}^{\infty}\eta^n B_{2n}e^{\theta_2} \tag{37b}$$

Substituting Eqs. (37) into Eqs. (35), equating the coefficients of exp (θ_i) to zero, and equating like powers of η, we obtain equations to determine successively A_{1n}, A_{2n}, B_{1n}, B_{2n}:

$$\left.\begin{aligned}\lambda_i A_{i0}+\alpha_1 B_{i0}=0\\ \lambda_i B_{i0}-\alpha_2 A_{i0}=0\end{aligned}\right\} \qquad (i=1,2) \tag{38}$$

$$\left.\begin{aligned}\lambda_i A_{i1}+\alpha_i B_{i1}=-\frac{d}{dh}A_{i0}\\ \lambda_i B_{i1}-\alpha_2 A_{i1}=-\frac{d}{dh}B_{i0}\end{aligned}\right\} \qquad (i=1,2) \tag{39}$$

From Eqs. (38)

$$B_{i0}=(\alpha_2/\lambda_i)A_{i0} \qquad (i=1,2) \tag{40}$$

Substitution of Eqs. (40) into Eqs. (39) and eliminating the terms that produce secular terms in Eqs. (39) yields

$$\frac{2}{A_{i0}}\frac{d}{dh}A_{i0}+\frac{1}{\alpha_2}\frac{d}{dh}\alpha_2-\frac{1}{\lambda_i}\frac{d}{dh}\lambda_i=0 \quad (i=1,2) \tag{41}$$

The solution of Eq. (41) is

$$A_{i0}=(|\lambda_i|^{1/2}/|\alpha_2|^{1/2})A_{i0C} \qquad (i=1,2) \tag{42}$$

where A_{i0C} is a constant. Then, from Eq. (40)

$$B_{i0}=(|\lambda_i|^{1/2}/|\alpha_2|^{1/2})(\alpha_2/\lambda_i)A_{i0C} \quad (i=1,2) \tag{43}$$

Therefore, to the first approximation, ω_1 and ω_2 are given by

$$\omega_1=(|\lambda_1|^{1/2}/|\alpha_2|^{1/2})A_{i0C}e^{\theta_1}+(|\lambda_2|^{1/2}/|\alpha_2|^{1/2})A_{20C}e^{\theta_2} \tag{44a}$$

$$\omega_2=(|\lambda_1|^{1/2}/|\alpha_2|^{1/2})(\alpha_2/\lambda_1)A_{10C}e^{\theta_1}+(|\lambda_2|^{1/2}/|\alpha_2|^{1/2})(\alpha_2/\lambda_2)A_{20C}e^{\theta_2} \tag{44b}$$

Here, it is worthwhile to note that these solutions are inapplicable near the turning points h_1 and h_2 $(h_1>h_2)$, which are given by $\alpha_1\alpha_2=0$. Equations (44) show that ω_1 and ω_2 are oscillatory in a region where $\alpha_1\alpha_2>0$, while they are exponential in a region where $\alpha_1\alpha_2<0$. In Fig. 3b, the quantities ω_1 and ω_2 behave oscillatory in a stable region (regions I and III), while they behave exponentially in an unstable region (region II).

Region I $(h_2<h)$:

$$\omega_1=|\alpha_1|^{1/2}/(|\alpha_1\alpha_2|)^{1/4}[a_1\cos(\theta_1+\pi/4)-b_1\sin(\theta_1+\pi/4)] \tag{45a}$$

$$\omega_2=|\alpha_2|^{1/2}/(|\alpha_1\alpha_2|)^{1/4}[a_1\sin(\theta_1+\pi/4)+b_1\cos(\theta_1+\pi/4)] \tag{45b}$$

$$\theta_1=\int_{h_2}(|\alpha_1\alpha_2|)^{1/2}\eta^{-1}dh \tag{45c}$$

Region II $(h_1<h<h_2)$:

$$\omega_1=|\alpha_1|^{1/2}/(|\alpha_1\alpha_2|)^{1/4}(a_2e^{\theta_2}+b_2e^{-\theta_2}) \tag{46a}$$

$$\omega_2=|\alpha_2|^{1/2}/(|\alpha_1\alpha_2|)^{1/4}(-a_2e^{\theta_2}+b_2e^{-\theta_2}) \tag{46b}$$

$$\theta_2=\int_{h_2}(|\alpha_1\alpha_2|)^{1/2}\eta^{-1}dh \tag{46c}$$

Region III $(h<h_1)$:

$$\omega_1=|\alpha_1|^{1/2}/(|\alpha_1\alpha_2|)^{1/4}[a_3\cos(\theta_3+\pi/4)-b_3\sin(\theta_3+\pi/4)] \tag{47a}$$

$$\omega_2=|\alpha_2|^{1/2}/(|\alpha_1\alpha_2|)^{1/4}[-a_3\sin(\theta_3+\pi/4)-b_3\cos(\theta_3+\pi/4)] \tag{47b}$$

$$\theta_3=\int_{h_1}(|\alpha_1\alpha_2|)^{1/2}\eta^{-1}dh \tag{47c}$$

Since these expressions are asymptotic forms of the same solution of a linear system (35), there must be linear relations between the coefficients (a_1, b_2) (a_2, b_2) and (a_3, b_3). These relations can be obtained by using the method of matched asymptotic expansions.[8] We shall first determine the relations between (a_1, b_1) and (a_2, b_2). We may calculate the relations by considering the solution of Eqs. (35) near turning point h_2. Near the turning point h_2, $\alpha_1=\alpha_1(h_2)$ and

$\alpha_2 = \alpha_{20}(h-h_2)$. Then, Eqs. (35) become

$$\dot{\omega}_1 + \alpha_1(h_2)\omega_2 = 0 \tag{48a}$$

$$\dot{\omega}_2 - \alpha_{20}(h-h_2)\omega_1 = 0 \tag{48b}$$

or

$$\ddot{\omega}_1 + \alpha_{20}\alpha_1(h_2)(h-h_2)\omega_1 = 0 \tag{49}$$

Introducing new variable y by the equation

$$y = (h-h_2)\eta^{-2/3} \tag{50}$$

we obtain

$$\frac{d^2}{dy^2}\omega_1 + \alpha_{20}\alpha_1(h_2)y\omega_1 = 0 \tag{51}$$

The solution of Eq. (51) is given by

$$\omega_1 = c_2A_i[-|\alpha_{20}\alpha_1(h_2)|^{1/3}y] + d_2B_i[-|\alpha_{20}\alpha_1(h_2)|^{1/3}y] \tag{52}$$

where $A_i(z)$ and $B_i(z)$ are the first and second kinds of Airy functions, respectively. For large y, the solution (52) has the following asymptotic expansions:

$$\omega_1 = |\alpha_{20}\alpha_1(h_2)|^{-1/12}y^{-1/4} \times \{(c_2/\sqrt{\pi})\sin[2/3|\alpha_{20}\alpha_1(h_2)|^{1/2}y^{3/2} + j/4] + (d_2/\sqrt{\pi})\cos[2/3|\alpha_{20}\alpha_1(h_2)|^{1/2}y^{3/2} + \pi/4]\} \quad (y>0) \tag{53}$$

$$\omega_1 = |\alpha_{20}\alpha_1(h_2)|^{-1/12}y^{-1/4} \times \{(c_2/2\sqrt{\pi})\exp[(-2/3)|\alpha_{20}\alpha_1(h_2)|^{1/2}|y|^{3/2}] + (d_2/\sqrt{\pi})\exp[(2/3)|\alpha_{20}\alpha_1(h_2)|^{1/2}|y|^{3/2}]\} \quad (y<0) \tag{54}$$

The solutions (45) and (46) are also expressed in terms of y in the form

Region I ($y>0$):

$$\omega_1 = \frac{|\alpha_1(h_2)|^{1/4}}{|\alpha_{20}\eta^{1/3}|^{1/4}}y^{-1/4} \times \{a_1\sin[(2/3)|\alpha_{20}\alpha_1(h_2)|^{1/2}y^{3/2} + \pi/4] - b_1\cos[(2/3)|\alpha_{20}\alpha_1(h_2)|^{1/2}y^{3/2} + \pi/4]\} \tag{55}$$

Region II ($y<0$):

$$\omega_1 = \frac{|\alpha_1(h_2)|^{1/4}}{|\alpha_{20}\eta^{2/3}|^{1/4}}|y|^{-1}\{a_2\exp[(2/3)|\alpha_{20}\alpha_1(h_2)|^{1/2}|y|^{3/2}] + b_2\exp[-(2/3)|\alpha_{20}\alpha_1(h_2)|^{1/2}|y|^{3/2}]\} \tag{56}$$

The matching principle demands that Eqs. (53) and (55), and Eqs. (54) and (56) be equal:

$$\frac{|\alpha_1(h_2)|^{1/4}}{|\alpha_{20}\eta^{1/3}|^{1/4}}a_1 = \frac{|\alpha_{20}\alpha_1(h_2)|^{-1/12}}{\sqrt{\pi}}c_2 = \frac{2|\alpha_1(h_2)|^{1/4}}{|\alpha_{20}\eta^{1/3}|^{1/4}}b_2 \tag{57a}$$

$$\frac{-|\alpha_1(h_2)|^{1/4}}{|\alpha_{20}\eta^{1/3}|^{1/4}}b_1 = \frac{|\alpha_{20}\alpha_1(h_2)|^{-1/12}}{\sqrt{\pi}}d_2 = \frac{|\alpha_1(h_2)|^{1/4}}{|\alpha_{20}\eta^{1/3}|^{1/4}}a_2 \tag{57b}$$

From Eqs. (57), we obtain the relations between (a_1, b_1) and (a_2, b_2):

$$b_2 = a_1/2 \qquad a_2 = -b_1 \tag{58}$$

Near the turning point h_1, the calculations can be carried out in an entirely similar manner; the relations between (a_2, b_2) and (a_3, b_3) are derived as follows:

$$a_3 = 2a_2e^{\Gamma_{21}} \qquad b_3 = b_2e^{-\Gamma_{21}} \tag{59}$$

where

$$\Gamma_{21} = \int_{h_2}^{h_1}(|\alpha_1\alpha_2|)^{1/2}\eta^{-1}dh$$

According to the above results, neglecting small quantities, we can express a_3 and b_3, the coefficients of the solution in region III, in terms of a_1 and b_1, the coefficients of the solution in region I:

$$a_3 = -2b_1e^{\Gamma_{21}} \qquad b_3 = 0 \tag{60}$$

The amplitude R_M of the residual nutational body motion during its passage through the region of instability, which is defined by $[(|\alpha_2/\alpha_1|)^{1/2}\omega_1^2 + (|\alpha_1/\alpha_2|)^{1/2}\omega_2^2]^{1/2}$ is given by

$$R_M = 2R_ie^{\Gamma_{21}} \tag{61}$$

where R_i is the amplitude of an initial nutational body motion. Figure 4 shows the amplitude R_M of residual nutational body motion as a function of Δ_A/I for some values of η; the system parameters are $I_{A3}/I=0.8$, $I_{B3}/I=0.4$, $H_0/I=1.0\ s^{-1}$, and $R_i=0.01$ rad/s:

1) $\eta/I = 0.003 \quad s^{-2}$
2) $\eta/I = 0.006 \quad s^{-2}$
3) $\eta/I = 0.009 \quad s^{-2}$
4) $\eta/I = 0.012 \quad s^{-2}$

Let us now consider the phenomena that occur when a nutational body motion begins to be amplified. The increase of a nutational body motion may have a considerable effect on the behavior of the system. Increase of nutational body motions leads to the appearance of a trap phenomenon

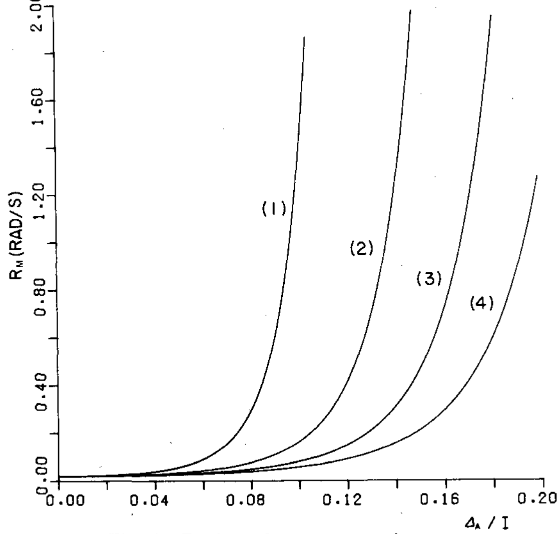

Fig. 4 Region of occurrence of a trap.

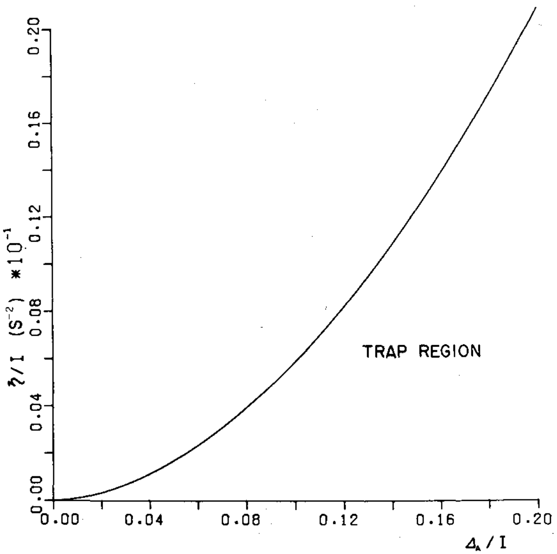

Fig. 5 Amplitude R_M of residual nutational body motion.

(inability to path through an unstable region). Substituting solution (46) in region II into Eq. (31), we obtain

$$\dot{\omega}_3 = -2(\Delta_A/I_{A_3})(a_2^2 e^{2\theta_2} - b_2^2 e^{-2\theta_2}) - \eta/I_{A_3} \tag{62}$$

where the first term on the right gives the reaction torque due to nutational body motions. When the right-hand side becomes negative, the angular velocity of body A ceases to increase in the course of time; i.e., the system is trapped in region II. The criterion under which the trap may occur is given by the formula

$$-(\Delta_A/I_{A_3})R_M^2 e^{2\Gamma_{21}}/2 - \eta/I_{A_3} < 0 \tag{63}$$

The limiting curve is shown in Fig. 5 for the case where $I_{A_3}/I = 0.8$, $I_{B_3}/I = 0.4$, $\Delta_I/I = 0.1$, $H_0/I = 1.0$ rad/s, and $R_i = 0.001$ rad/s. Finally, let $\eta > 0$; the system starts at some point in region III and reaches region I.

Repeating the calculations given above, we have the solutions to the problem. The amplitude R_M of residual nutational body motions is given by

$$R_M = 2R_i e^{\Gamma_{12}} \tag{64}$$

where

$$\Gamma_{12} = \int_{h_1}^{h_2} (|\alpha_1 \alpha_2|)^{1/2} \eta^{-1} \mathrm{d}h$$

The trap condition can be given by

$$-(\Delta_A/I_{A_3})R_M^2 e^{2\Gamma_{12}}/2 - \eta/I_{A_3} > 0 \tag{65}$$

This condition does not hold; in this case the trap phenomenon does not occur. In order to compare these results, numerical investigation has been made on the basis of Eqs. (29-32). Figures 3 show some examples of the attitude behavior in passing through the unstable region. In the case where a driving torque is sufficiently large, the spacecraft can pass through the unstable region (cases 1, 2). On the other hand, when an undersized torque motor is used, the spacecraft cannot pass through the unstable region; i.e., a trap occurs (case 3).

Conclusion

The attitude behavior of a dual-spin spacecraft composed of asymmetric bodies has been investigated. It has been clarified that this class of spacecraft has an unstable region due to the asymmetry of the bodies. Nonstationary attitude motions in passing through the unstable region have been examined by an analytical method that utilizes the WKB method. It has been found that, under certain conditions, a trap phenomenon may occur. In order to avoid the trap phenomenon, the following practices are recommended:

1) If the transverse moments of inertia of a rotor differ significantly, a powerful motor should be used.

2) A nutational body motion should be completely damped at the beginning of the despin maneuver.

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